

Orthogonal Collocation Revisited

by Larry C. Young

After having been involved during the early days of orthogonal collocation [Young and Finlayson, 1973, 1976], I had reason to use it again recently. A review of the last 35 years revealed that some aspects of the method appear to have never been explained clearly or never explained at all. This realization prompted the publication of this note.

Here we first lay the fundamental framework for the method. Then, we show what works and what does not work using a series of examples. In many cases the examples are solved not only with orthogonal collocation, but also with other methods for comparison, e.g. Galerkin, moments and finite differences. The examples also cover a variety of problems: boundary value problems, parabolic equations, hyperbolic equations, one and two spatial dimensions. The examples show the advantages and disadvantages of the method for each type of problem.

We look not only at global methods, where the solution is approximated by a single polynomial, but also finite element methods. With finite elements, polynomials are pieced together to approximate the solution.

The following is an outline of this discussion:

1. Basis of the method
2. Selection of trial functions
3. Approximation of integrals
4. Diffusion with Reaction (nonsymmetric)
 - Orthogonal Collocation Method
 - Moments Method
 - Galerkin Method
5. Mass Conservation and Fluxes
6. Diffusion with Reaction (symmetric)
 - geometry and third kind boundary conditions
 - constant diffusion
 - nonlinear reaction
 - nonlinear diffusion
7. Reaction with Axial Dispersion
8. Graetz Problem in Round Tube
9. Damped Spring with Specified End Motion
 - Orthogonal Collocation Method
 - Galerkin Method
 - Finite Difference Method
10. Graetz Problem in Rectangular Duct
11. Convective Diffusion Problem
 - Finite Elements with Gauss Points
 - Finite Elements with Lobatto Points
 - Galerkin Method
 - Finite Difference Method

An Appendix describes how to calculate the matrix operators for approximating integrals and derivatives with all of these methods.

A computer code for calculating the matrix operators is available on the internet. Computer codes (in Fortran 90, C++ and Excel) for solving most of the example problems are also available. The computer codes make this work a convenient tutorial for learning about orthogonal collocation and related methods for the numerical solution of various types of differential equations.