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Triple-Curve Space Puts

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THE conventional contour for a cam intended for use with a mushroom tappet—composed of a circular-arc nose and a circular-arc flank—is far from ideal when space limitations make it impossible to use a tappet of sufficient diameter to obtain the required lift. Lift decreases in almost direct proportion with the permissible tappet diameter. Mathematical analysis shows how this deficiency can be corrected in a very simple manner.

Analysis of the Conventional Cam

Fig. 1-a shows an ordinary mushroom cam in contact with a flat follower at that point of its contour which marks the end of the flank and the beginning of the nose. It is at this point that acceleration ceases and deceleration begins, and it is therefore generally referred to as the point of reversal. The angular position of the cam for motion of the tappet in contact with the nose is defined by the angle Θ, which is measured from the apex of the nose, and that for motion on the flank by the angle ϕ, which is measured from the beginning of the lift, where the flank arc meets the cam base circle.

From the geometry of Fig. 1-a we arrive at an expression for the lift. For motion of the tappet in contact with the nose we can write—

$$\text{Fall} = E_n(1 - \cos\theta),$$

and for motion in contact with the flank—

$$\text{Lift} = E_{fl}(1 - \cos\phi).$$

$$V_n = \frac{dL}{dt} = E_n \sin\theta \frac{d\theta}{dt} \quad \text{(on nose)}$$

$$V_{fl} = \frac{dL}{dt} = E_{fl} \sin\phi \frac{d\phi}{dt} \quad \text{(on flank)}$$

In these equations $d\theta/dt$ and $d\phi/dt$ are equal to the angular velocity of ω of the cam which is constant and is expressed in radians per second. Therefore,

$$V_n = E_n\omega \sin\theta \quad \text{(on nose)}$$

$$V_{fl} = E_{fl} \sin\phi\,\omega \quad \text{(on flank)}$$

$$\text{Acceleration} = \frac{dV}{dt} \text{ or,}$$

$$A_n = E_n\omega^2 \cos\theta \quad \text{(on nose)}$$

$$A_{fl} = E_{fl}\omega^2 \cos\phi \quad \text{(on flank)}$$

For unit angular velocity (one radian per second), corresponding to $30/\pi$ r.p.m., the expressions for velocity and acceleration become:

$$V_n = E_n \sin\theta \quad \text{(on nose)}$$

$$V_{fl} = E_{fl} \sin\phi \quad \text{(on flank)}$$

$$A_n = E_n \cos\theta \quad \text{(on nose)}$$

$$A_{fl} = E_{fl} \cos\phi \quad \text{(on flank)}$$

A study of these expressions reveals the following important facts:

(1) The tappet motion is analogous to the motion of a piston derived from a crank rotating at uniform angular velocity through the intermediary of a connecting rod of infinite length. Hence the conclusion that the tappet motion is made up of portions of two simple harmonic cycles of different amplitude, on the flank and the nose of the cam respectively.

(2) The flank eccentricity E_{fl} and the nose eccentricity E_n represent the respective crank throws and are the only basic cam dimensions. In other words, changing the nose radius or the flank radius without changing the nose and flank eccentricities has no effect on the characteristics of the cam.

(3) If we make the total deflection of the valve spring equal to the nose eccentricity E_n, the spring load is a constant function of the deceleration of the tappet motion on the nose. This can be proven as follows:

We have seen that

$$F_n = E_n(1 - \cos\theta) \quad \text{and}$$

$$A_n = E_n \cos\theta$$

For $\theta = 0$ degrees

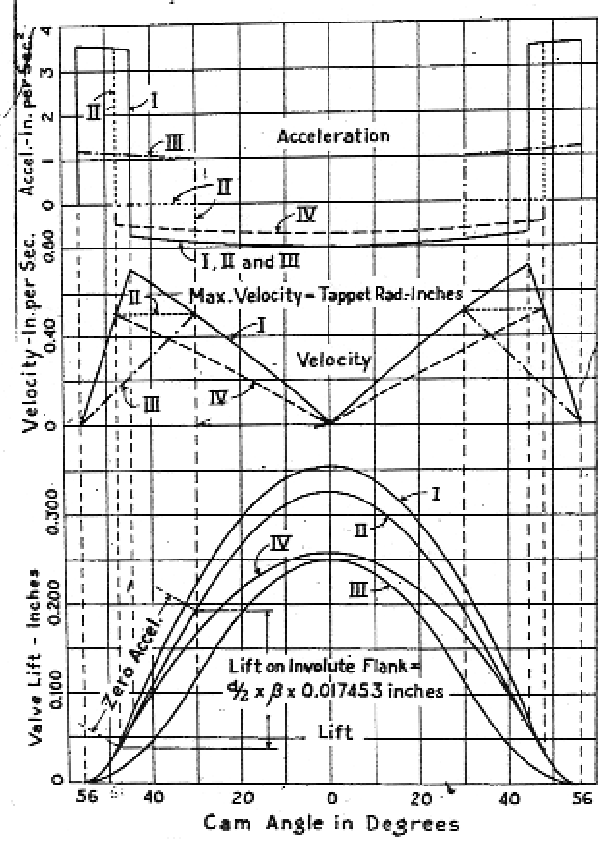


Fig. 1—Lift, velocity and acceleration curves for four cams for use with flat followers. Cams I, III and IV are similar and are illustrated by Fig. 1-a, while cam II, which first gives an upward acceleration to the follower, then maintains it at constant speed, and finally allows it to decelerate, is illustrated in Fig. 1-b

Cam Gives Maximum Lift Where Limit on Tappet Head Diameter

How to get maximum lift for a given tappet head diameter is a problem engineers are sometimes called upon to solve when the engine design limits this dimension, as may happen, for example, in a Vee engine. The tappet head, of course, must have sufficient diameter always to be tangent to the cam outline so that the edge of the tappet will not dig into the cam.

A triple-curve cam, instead of the usual double-curve type employed with mushroom followers, offers a solution of this problem as it permits use of a tappet head of smaller diameter. This cam has first an outline giving constant, rapid acceleration to the valve up to its maximum speed, then an involute form maintaining the upward speed of the valve constant for a period, and finally a circular arc nose which decelerates the valve until its upward speed is zero.

The accompanying article explains the design of such a cam.

$$F_n = 0 \qquad \text{and } A_n = E_n$$

For $\theta = 90$ degrees,

$$F_n = E_n \qquad \text{and } A_n = 0$$

If, instead of measuring the fall from the top of the nose, we measure the lift from the position corresponding to $\theta = 90$ deg., we can write:

$$L_n = E_n \cos\theta \qquad \text{and}$$

$$A_n = -E_n \cos\theta \qquad \text{so}$$

$$L_n = -A_n.$$

That is, the lift in inches is equal to the deceleration in inches per second per second for any value of Θ.

For $\theta = 90$ degrees

$$L_n = 0 \qquad \text{and } Dec_n = 0$$

For $\theta = 0$ degrees

$$L_n = E_n \qquad \text{and } Dec_n = E_n$$

The required spring load is zero for $\Theta = 90$ deg., and maximum at the top of the nose when $\Theta = O$, when the compression of the spring equals E_n, which in turn must be equal to the total deflection of the valve spring. By "total deflection of the valve spring" is meant, of course, the difference between its free length and the length of the spring when the valve is fully open.

The smaller the total deflection of the spring is made, the higher will be the natural rate of vibration, and for that reason the total deflection is usually made considerably less than the nose-eccentricity. In that case the spring must be designed to overcome the inertia force at the point of reversal, rather than at the top of the nose.

(4) Maximum velocity is reached at the point of reversal, since this point corresponds to the maximum value of θ on the nose as well as the maximum value of ϕ on the flank, and we can write:

$$E_{fl} \sin \phi_{max.} = E_n \sin \theta_{max.}$$

From the geometry of Fig. 1-a it also follows that the distance from the point of contact between cam and follower, to the center line of the tappet is equal to

$$\frac{d}{2} = E_{fl} \sin \phi,$$

which becomes a maximum for the maximum value of ϕ (point of reversal), hence represents the minimum theoretical tappet radius in inches, and, incidentally, is equal to the maximum velocity in inches per second.

To analyze the possibilities of obtaining a high valve lift, with the limitations as to maximum acceleration, maximum deceleration and maximum velocity, we will reverse our method of computation. Instead of starting out with the expression for the lift, and by differentiation obtaining values for velocity and acceleration, we will start with the acceleration and find the expressions for velocity and lift by integration with respect to time. Representing the acceleration by a, we can write

$$V = \int a\, dt$$

$$V = \int a\, d\phi \frac{dt}{d\phi} = \frac{1}{\omega}\int a\, d\phi$$

For an angular velocity equal to one radian per second we substitute 1 for ω and get

$$V = \int a\, dt = \int a\, d\phi$$

in other words, the time scale and the angle scale are interchangeable.

Likewise,

Cam	En	Efl	d/2
I	1	3.5	0.70
II	1	3.5	0.50
III	1	1.14	0.50
IV	0.675	3.5	0.50

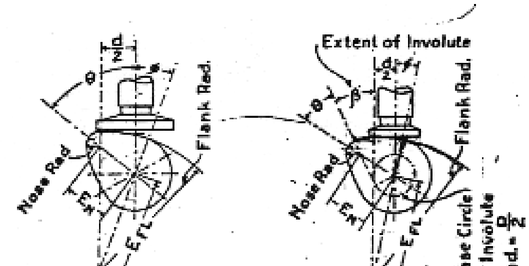
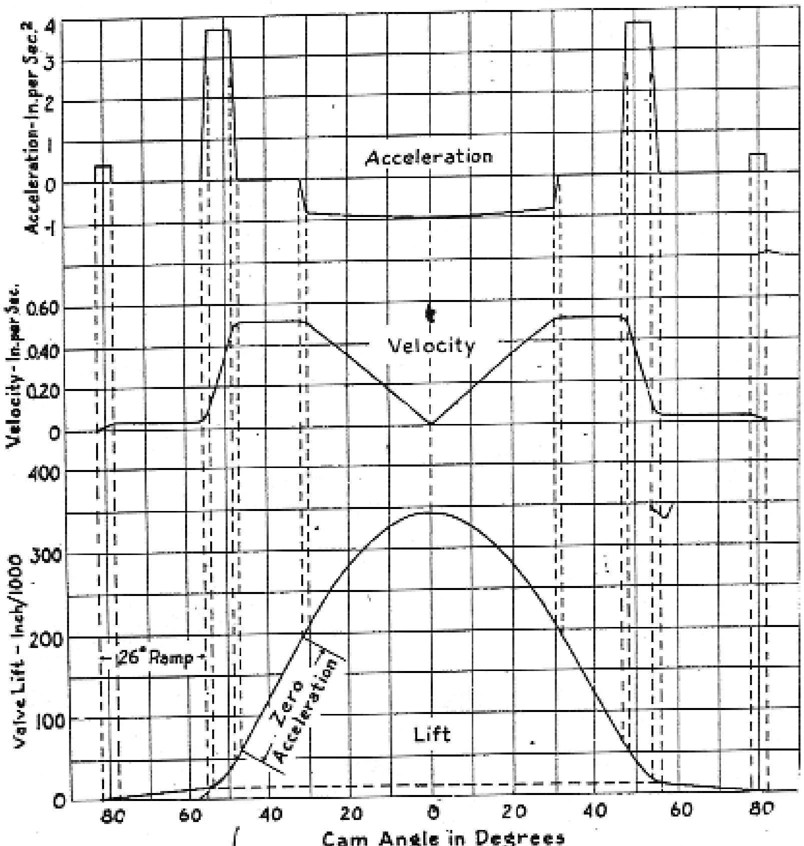


Fig. 1A—(Left) represents cams I, III, and IV, which are similar; Fig. 1b—(right) represents cam II



Angle	Lift	Angle	Lift	Angle	Lift
0	0.3400	28	0.2229	56	0.0120
2	0.3394	30	0.2060	58	0.0110
4	0.3376	32	0.1882	60	0.0100
6	0.3345	34	0.1703	62	0.0090
8	0.3304	36	0.1525	64	0.0080
10	0.3248	28	0.1346	66	0.0070
12	0.3181	40	0.1168	68	0.0060
14	0.3103	42	0.0990	70	0.0050
16	0.3012	44	0.0811	72	0.0040
18	0.2911	46	0.0633	74	0.0030
20	0.2797	48	0.0455	76	0.0020
22	0.2671	50	0.0306	78	0.0010
24	0.2535	52	0.0201	80	0.00025
26	0.2388	54	0.0140	82	0.00000

Fig. 2—Sample design of cam for use with flat follower of restricted diameter, the cam flank comprising an involute portion which raises the valve at constant speed

$$L = \int v\,dt = \int v\,d\phi$$

In geometrical terms, these equations mean that—

The velocity at ϕ_1 is equal to the area under the acceleration curve between zero and ϕ_1.

The lift at ϕ_1 is equal to the area under the acceleration curve between zero and ϕ_1.

The total lift is equal to the total area under the velocity curve.

In other words, to obtain the maximum possible lift, we must try to make the area under the velocity curve as large as possible. The maximum velocity is limited to the value of the tappet radius, which clearly indicates that there should be a period of constant velocity, which should be as long as possible.

Reverting to the expressions for the velocity on the conventional harmonic cam, we can write

$$\sin\phi = \frac{V_{fl}}{E_{fl}} \qquad \text{or,}$$

$$\sin\phi_{max} = \frac{d/2}{E_{fl}} \qquad \text{(on flank)}$$

Likewise on the nose—

$$\sin\theta = \frac{V_n}{E_n} \qquad \text{or,}$$

$$\sin\theta_{max} = \frac{d/2}{E_n} \qquad \text{(on nose)}$$

If the tappet radius $d/2$, and the flank-eccentricity E_{fl} are given, we can find the values of ϕ_{max} and Θ_{max} in degrees from these equations. Subtracting these values from half the cam angle ($= \alpha$ deg.) gives the duration of the required period of constant velocity (β) in degrees. The velocity during this period is equal to $d/2$ in inches per second, acceleration is zero, and the lift per second or per radian also is equal to $d/2$ in inches.

$$\beta = \alpha - (\phi_{max} + \theta_{max})$$

For the total lift we can write

$$L_{total} = L_{fl} + L_n + L_\beta$$

or

$$L_{total} = E_{fl}(1 - \cos\phi_{max}) + E_n(1 - \cos\theta_{max}) + d/2 \times 0.0174533 \text{ in inches,}$$

0.0174533 being the conversion factor from degrees to radians.

We will now figure out an example, and will calculate the cam shown in Fig. 1 (Cam II).

Here,

$$\alpha = 56 \text{ degrees}$$
$$E_{fl} = 3.5 \text{ in.}$$
$$E_n = 1 \text{ in.}$$
$$d/2 = 0.5 \text{ in.}$$
$$\sin\theta_{max} = \frac{d/2}{E_n} = \frac{0.5}{1} = 0.5$$
$$\theta_{max} = 30 \text{ degrees} \qquad \text{(on nose)}$$
$$\sin\phi_{max} = \frac{d/2}{E_{fl}} = \frac{0.5}{3.5} = 0.14286$$
$$\phi_{max} = 8°\,13' \qquad \text{(on flank)}$$
$$\beta = \alpha - (\theta_{max} + \phi_{max}) = 56° - (30° + 8°\,13')$$
$$\beta = 17°\,47'$$

(1) Lift on flank

$$L_{fl} = E_{fl}(1 - \cos\phi_{max})$$
$$L_{fl} = 3.5\,(1 - \cos 8°\,13')$$
$$L_{fl} = 3.5 \times 0.01025 = 0.0359 \text{ in.}$$

(2) Lift on nose.

$$L_n = E_n(1 - \cos\theta_{max})$$
$$L_n = 1\,(1 - \cos 30°)$$
$$L_n = 1 - 0.866$$
$$L_n = 0.1340 \text{ in.}$$

(3) Lift during period of constant velocity.

$$L_\beta = d/2 \times \beta \times 0.0174533$$
$$L_\beta = 0.5 \times 17°\,47' \times 0.0174533$$
$$L_\beta = 0.1552 \text{ in.}$$
$$\textit{Total Lift} = L_{fl} + L_n + L_\beta$$
$$= 0.0359 + 0.1340 + 0.1552$$
$$= 0.3251 \text{ in.}$$

It remains only to construct the cam profile. The nose eccentricity is taken as 1 in., and if we assume the nose radius to be 0.050 in., the base radius will be

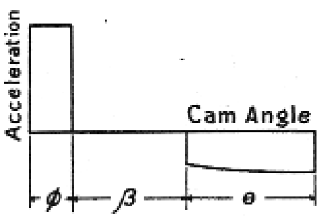


Fig. 3—Ideal acceleration diagram for cam designed for use with flat follower of restricted diameter

$$1.050 - \text{total lift, or}$$
$$1.050 - 0.325 = 0.625 \text{ in.}$$

After setting off half the cam angle (56 deg.) we draw the flank arc, which has a radius of 4.125 in. We next draw the nose arc, which extends over an angle of 30 deg. With the tappet diameter as a base circle we now draw an involute curve, starting at the end of the nose arc. This involute curve will join the flank arc at the calculated angle of 8 deg. 13 min.

This involute curve will meet the requirement of constant velocity for the tappet motion, equal in magnitude (numerically) to the tappet radius, because the normal at the point of contact between cam and follower will always be tangent to the base circle of the involute, which is made equal to the tappet diameter.

Comparison of Cam Characteristics

Four different cams with their resulting lift curves, velocity curves and acceleration curves are shown in Fig. 1. Assuming a maximum permissible flank eccentricity of 3.5 in. and a maximum permissible nose eccentricity of 1 in., cam I is laid out regardless of tappet diameter. The other cams are laid out for a tappet radius of 0.5 in. The diagrams clearly show the loss in lift due to a reduction in the tappet radius, as well as the improvement due to the constant-velocity period in cam II.

Cam III is laid out with the maximum permissible nose [illegible] 1 in. With this nose [illegible] a flank [illegible] of 1¼ in. is necessary in order not to exceed the maximum allowable velocity of 0.5 in. per second.

Cam IV is laid out with the maximum permissible flank [illegible] of 3.5 in., which results in a nose [illegible] of 0.675 in. Of the two, cam IV is the best, but it compares very unfavorably with cam II with the constant-velocity feature.

The total lift of cam IV is slightly greater than that of cam III, although the maximum velocity is the same for both. This is easily explained if we look at the velocity diagrams of the two cams. We know that the total lift is equal to the area of the velocity diagram, and although both diagrams are triangular in shape and have the same base and the same height, their sides are not straight lines, and the areas therefore differ.

It will be obvious that the smaller we can make the angles ϕ_{max} and Θ_{max}, the larger the angle β and the higher the resulting total lift will be. To make the angles ϕ_{max} and Θ_{max} small, we should make both acceleration and deceleration as high as is practically possible.

The maximum acceleration is limited by the allowable maximum tappet load; the maximum deceleration is equal to the nose radius and therefore limited to half the camshaft bearing diameter minus the nose radius. To avoid manufacturing difficulties we should not go too far with reducing the nose radius; 0.050 in. can be set down as a reasonable minimum.

For that reason, a change in contour of the nose, so as to obtain constant deceleration, with the object of reducing the duration of deceleration while giving the same maximum velocity, would not give the desired results. The limit of 0.050 in. radius of curvature set for the profile of the nose, means that we cannot increase the deceleration at

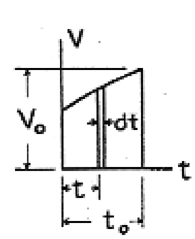


Fig. 4—Valve-velocity diagram

the point of Θ_{max} above the deceleration obtained with the circular-arc-nose cam.

On the flank, however, we have no such limitations, and even though the gain in total lift is small, there does not appear to be any objection to changing the contour so as to obtain constant acceleration. An added advantage can be found in the simple expressions for the velocity and for the lift, which become:

$$\text{Velocity} = at = a\theta$$

$$\text{Lift} = 1/2\, a\, t^2 = 1/2\, a\, \theta^2 \quad (\theta \text{ in radians})$$

or $$\text{Velocity} = 0.0174533\, a\theta$$

$$\text{Lift} = 0.0174533^2 \times 1/2\, a\theta^2 \quad (\theta \text{ in deg.})$$

Fig. 3 shows this ideal acceleration diagram, which consists of three distinct periods.

(1) Period of constant acceleration.
(2) Period of zero acceleration.
(3) Period of deceleration according to the expression

$$\text{Dec.} = E_n \cos \theta$$

The maximum value for θ can easily be found, and the rest of the computation is identical with that given for the cam with circular-arc flank. Construction of the actual cam profile for constant acceleration is rather difficult, but is not necessary. All we are really interested in is the lift curve and a tabulation of the lift per degree rotation, which is all that is necessary to make the master cam.

Quieting Curve or "Ramp"

So far, the question of the quieting curve or "ramp" has been neglected in the analysis. The ramp represents a period of low, constant velocity and zero acceleration, and to reach this velocity, should be preceded by a period of acceleration—preferably of constant acceleration—of short duration. This means, of course, that the cam profile for this period of constant velocity follows an involute curve, while the radius of the base circle of this involute in inches is equal to the velocity on the ramp in inches per second, for an angular velocity of one radian per second ($30/\pi$ r.p.m.). To avoid a change in tappet velocity when leaving the ramp and beginning contact with the flank, the ramp should join the flank at that point where the velocity on the flank equals the ramp velocity.

The lift for the motion on the ramp over the first period of constant acceleration follows a parabolic curve. If the velocity at the end of this period is 0.0005 in. per deg., the duration of this period four degrees, then:

Lift	first degree	= 0.0000625	in.
	second degree	= 0.000250	in.
	third degree	= 0.000625	in.
	fourth degree	= 0.001000	in.

From there on, up to the point where the ramp ends, a rate of lift of 0.0005 in. per deg. should be maintained.

We have seen that the area under the acceleration curve must equal the area under the deceleration curve. Addition of a ramp means, of course, an additional area of acceleration diagram for the period of constant acceleration preceding the ramp, hence requires a reduction by the same amount of the area of the acceleration diagram for the flank. This is automatically taken care of if the ramp involute joins the flank arc at the point of equal velocity, as

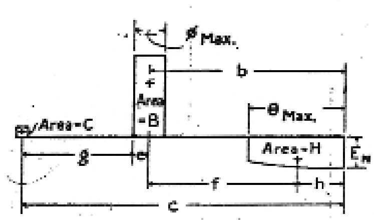


Fig. 5—Diagram serving to demonstrate that the lift is equal to the static moment of the acceleration diagram

mentioned above. The location of this point can be easily found.

The expression for the velocity on the flank is:

$$V_{f1} = E_{f1} \sin \phi$$

Substituting for V_{f1} the ramp velocity will give the solution for ϕ.

For a ramp velocity of 0.0005 in. per deg. or 0.0005 $\times$ 0.0174533 in. per radian, and E_{f1} equal to 3.5 in., the angle ϕ is equal to 27 min. For the constant-acceleration-flank cam this angle will be slightly different, of course. The expression for the velocity is $V = a\ \phi$, and this can be solved for ϕ.

In the computation the ramp should be taken care of right from the start, rather than later on. If the tappet radius and, therefore, the maximum velocity is given, we subtract from this value the ramp velocity (in inches per radian) and use this figure as the required area of the acceleration diagram of the flank.

Lift equals static moment of acceleration diagram

We have seen that the velocity is equal to the area of the acceleration diagram, and that the lift is equal to the area of the velocity diagram, and will now prove that the lift is also equal to the static moment of the acceleration diagram. This makes it possible to compute the lift at any point of the cycle directly from any arbitrary acceleration diagram.

Computation of the lift curve is particularly simple if the acceleration diagram is of such a shape that the center of gravity as well as the area can be easily found. Referring to Fig. 4,

$$L_{t_o} = \int_o^{t_o} v\, dt$$

$$= v_o\, t_o - \int_o^{v_o} t\, dv$$

$$a = \frac{dv}{dt},$$

so

$$L_{t_o} = v_o\, t_o - \int_o^{t_o} a\, t\, dt$$

$$= t_o \int_o^{t_o} a\, dt - \int_o^{t_o} a\, t\, dt$$

$$= \int_o^{t_o} a\, dt\, (t_o - t)$$

= Static Moment

This means that the lift at t_o is equal to the static moment with respect to t_o, or equal to the area of the acceleration diagram times the distance of the center of gravity from t_o.

For Fig. 5 we can write:

$$\text{Total lift} = bB + cC \overset{-}{+} hH \quad \text{or,}$$
$$= b(H-C) + cC - hH$$
$$= (b-h)\, H + (c-b)\, C$$
$$= f\, H + (g+e)\, C$$
$$= f \times \text{tappet radius} + (g+e) \times \text{ramp velocity.}$$

It will be clear that the lift at any point of the lift curve can be found in a similar manner. For a circular-arc-nose cam, the value of h in Fig. 5 can be easily calculated, since

$$F_n = E_n (1 - \cos \theta_{max})$$

and

$$F_n = H\, (\theta_{max} - h)$$

Knowing that H is equal to the maximum velocity and equal to the tappet radius, we can solve for h.

The lift curve of Fig. 2 was calculated by this method, and the results were tabulated. To avoid a sudden application of inertia load, the acceleration in Fig. 2 is shown starting with zero but rising to its maximum value rapidly.

Although it may be of little practical value, it is interesting to note that the area of the lift diagram is also equal to the moment of inertia of the acceleration diagram.

Automotive Oddities—By Pete Keenan